Is it possible to improve the weighting function for lightness in the CIEDE2000 color-difference formula?

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ABSTRACT

We have compared the performance of the CIEDE2000 color-difference formula (ΔE_{00}) with three CIEDE2000-modified formulas: 1) ΔE_{00}_M1, which incorporates a new V-shaped function proposed at the University of Leeds (UK) with a minimum at the specific lightness of the background; 2) ΔE_{00}_M2, a formula where the original S_L function in ΔE_{00} was replaced by $S_L=1$, as proposed by the CIE94 color-difference formula; 3) ΔE_{00}_M3, a formula developed by us, with the same structure than ΔE_{00}, but avoiding its original S_L function by replacing the lightness differences in CIELAB by a new lightness definition based on Whittle formula. Our comparison used the STRESS index and thirteen visual datasets (CIE 217:2016), including filtered subsets to test the symmetry of the S_L function proposed by ΔE_{00}. None of the three mentioned CIEDE2000-modified formulas performed statistically significantly better than the original ΔE_{00} formula for any of the mentioned datasets or subsets, with only one exception (ΔE_{00}_M2 formula, Witt dataset). Therefore, the replacement of the S_L function in ΔE_{00} by $S_L=1$ is not recommended. ΔE_{00}_M1 and ΔE_{00}_M3 improved ΔE_{00} for most datasets, but such improvements were not statistically significant. Results for color pairs with average L^* values below and above 50 were not statistically significant different for neither ΔE_{00}, ΔE_{00}_M2 and ΔE_{00}_M3 formulas. It is interesting to note that for eight of the thirteen visual datasets there were no color pairs with average L^* values below 25, which claims for future studies using darker color pairs.

KEYWORDS

Color difference, Color-difference formula, CIE94, CIEDE2000, Whittle formula

Received 16 July 2018; **Revised** 13 August 2018; **Accepted** 07 September 2018

CITATION: Melgosa M., Cui G., Oleari C., Pardo P.J., Huang M., Li C., Luo M. R. (2018) 'Is it possible to improve the weighting function for lightness in the CIEDE2000 color-difference formula?', Cultura e Scienza del Colore - Color Culture and Science Journal, 10, pp. 59-65, DOI: 10.23738/ccsj.i102018.07

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1. INTRODUCTION

Amongst the five corrections to CIELAB proposed by the current CIE-ISO recommended color-difference formula CIEDE2000 (ISO/CIE, 2014; CIE, 2001; Luo, Cui and Rigg, 2001), it can be said that the weighting function for lightness (symbol S_L), also called lightness tolerance, has been the most controversial one in recent literature (Melgosa et al., 2017). The CIEDE2000 color-difference formula (symbol ΔE_{00}), proposed a V-shaped symmetrical function S_L with a minimum at L^*=50 (the assumed lightness of the background), accounting for the so-called 'crispening effect'. In the current paper, from 13 experimental datasets (7420 color pairs) previously employed by the CIE Technical Committee 1-55 (CIE, 2016), we have used the Standardized Residual Sum of Squares (STRESS) index (García et al., 2007) to test the performances of the S_L function proposed by CIEDE2000, as well as three CIEDE2000-modified color-difference formulas. Low STRESS values, always in the range 0-100, indicate better performance of a color-difference formula (i.e. better predictions of average visually-perceived color differences reported by real observers with normal color vision).

The CIEDE2000 color-difference formula (ISO/CIE, 2014; CIE, 2001; Luo, Cui and Rigg, 2001) is given by:

$$\Delta E_{00} = \sqrt{\left(\frac{\Delta L'}{k_L S_L}\right)^2 + \left(\frac{\Delta C'}{k_C S_C}\right)^2 + \left(\frac{\Delta H'}{k_H S_H}\right)^2 + R_T\left(\frac{\Delta C'}{k_C S_C}\right)\left(\frac{\Delta H'}{k_H S_H}\right)}, \quad (1)$$

where the three parametric factors will be assumed in this paper as k_L=k_C=k_H=1.0 (i.e. the so-called 'reference conditions'), and the weighting function for lightness, S_L, is given by:

$$S_L = 1 + \frac{0.015\left(\overline{L^*} - 50\right)^2}{\sqrt{20 + \left(\overline{L^*} - 50\right)^2}} \quad . (2)$$

It can be noted that the CIEDE2000 color-difference formula in Eq. 1 is not an Euclidean distance, because of the chroma-hue interaction term controled by the R_T factor, which is often designated in the literature as the rotation term. The following three CIEDE2000-modified color-difference formulas will be considered in this paper:

1) The ΔE_{00}_M1 color-difference formula, analogous to ΔE_{00} in Eq. 1, but using the next S_L' function, proposed by researchers at the University of Leeds (UK) (Ho, 2006):

$$S_L' = 1 + \frac{0.008\left(\overline{L^*} - L_b\right)^2}{\sqrt{10 + \left(\overline{L^*} - L_b\right)^2}} \quad , (3)$$

where L_b is the specific CIELAB lightness of the background for each visual dataset.

2) The ΔE_{00}_M2 color-difference formula, also analogous to ΔE_{00}, but replacing the S_L function in Eq. 2 by $S_L = 1$, as proposed by the CIELAB and CIE94 (CIE, 2004) color-difference formulas. In a recent paper, R.S. Berns has recommended to introduce this modification in CIEDE2000 (Berns, 2016).

3) Finally, ΔE_{00}_M3 is a color-difference formula recently developed by us (Melgosa et al., 2017), with the same structure than ΔE_{00}, but avoiding its original S_L function. Specifically, the ΔE_{00}_M3 color-difference formula proposes the replacement of the ratio $\Delta L^*/S_L$ in Eq. 1 by ΔL_w, where L_w is the next new definition of lightness, based on Whittle formula (Whittle, 1992):

$$If\ Y \geq 18.419\ then\ L_w = $$
$$= 50.4\ log_{10}\left(1 + 1.21\ \frac{Y - 18.419}{4.73 + 18.419}\right) + 38.141 \quad (4)$$

$$If\ Y \leq 18.419\ then\ L_w = $$
$$= -50.4\ log_{10}\left(1 + 1.21\ \frac{18.419 - Y}{4.73 + Y}\right) + 38.141 \quad (5)$$

where Y is the luminance factor in the range 0-100; i.e. relative colorimetry (CIE, 2004).

For each one of the 13 visual datasets mentioned before, in addition to the complete datasets we have also considered different subsets: Color pairs with almost only lightness differences (specifically, color pairs with $|\Delta L^*/\Delta E^*_{ab}| > 0.9$), where ΔE^*_{ab} is the color-difference in CIELAB units), and color pairs with average L^* values below and above 50, the assumed lightness of the background in ΔE_{00} (see Eq. 2). Table 1 shows the percentage of color pairs in five different subsets, for each of the 13 visual datasets considered here. Excluding the two 'region specific' datasets with only blue (Lee et al., 2011) and black (Shamey et al., 2014) color pairs, Table 1 shows that the percentage of color pairs below and above $\overline{L^*}$=50 is balanced well enough (averages of 42.2% vs. 57.8%). However, it is noticeable that the percentage of color pairs with $\overline{L^*}$≤25 is considerably low: More specifically, Table 1 shows that there were no color pairs with $\bar{L}^*\leq 25$ for 9 of the 13 tested datasets, and the percentage in this range was 100% for the NCSU-Black (Shamey et al., 2014) dataset.

2. RESULTS

2.1. COMPLETE DATASETS

Table 2 shows STRESS values for each of the 13 visual datasets and different color-difference formulas. Specifically, Table 2 shows the STRESS results (García et al., 2007; CIE, 2016) for the original ΔE_{00} color-difference formula in column 2, three modifications of this formula previously described (ΔE_{00}_M1, ΔE_{00}_M2 and ΔE_{00}_M3) in columns 3-5, and the results found by the original ΔE_{00} formula modified by a power function with exponent 0.70 (column 6), which produced particularly good results in a previous work (Huang et al., 2015). The last two columns in Table 2 are the critical F-values to be considered at a 95% confidence level in order to test the statistical significance of the differences between two given color-difference formulas (García et al., 2007; CIE, 2016). Cells filled with blue/yellow color in columns 3-6 of Table 2 mean that the formula indicated in the header row performs better/worse than the original ΔE_{00} formula. In addition, numbers in bold/italic fonts in columns 3-6 of Table 2 indicate that there were statistically significant/non-significant differences (95% confidence level) between the formula in the header row and the original ΔE_{00} formula.

From Table 2 it can be noted that for the three CIEDE2000-modified formulas (columns 3-5) there is only one case with statistically significant better performance than the original ΔE_{00} formula: The case of the ΔE_{00}_M2 color-difference formula in Witt's dataset (Witt, 1999), with a cell filled with blue color and a STRESS value of 27.4 in bold font. This situation is in contrast with the highly satisfactory results achieved by using a power function with exponent 0.70 in ΔE_{00} (Huang et al., 2015), which produced statistically significant improvements upon the original ΔE_{00} formula for 9 of the 13 datasets, as indicated by the cells filled in blue color with numbers in bold font in column 6 of Table 2. Currently, it can be said that the use of power functions is the most effective way to improve the performance of advanced color-difference formulas (Huang et al., 2015).

2.2 COLOR PAIRS WITH MAINLY LIGHTNESS DIFFERENCES

Table 3 shows analogous results to those in previous Table 2, but filtering each one of the original datasets to consider only the color pairs with $|\Delta L^*/\Delta E^*_{ab}|>0.9$ (i.e. color pairs with almost only lightness differences). Because we are interested on studying the weighting function for lightness in ΔE_{00}, results shown in Tables 2 and 3 allow us to check whether the performances of ΔE_{00} and the remaining proposed color-difference formulas are or not different for the complete datasets (Table 2) and their corresponding filtered subsets with color pairs exhibiting almost only lightness

Datasets (References)	$\lvert\Delta L^*/\Delta E^*_{ab}\rvert > 0.9$	$\bar{L}^* \leq 25$	$\bar{L}^* \leq 50$	$\bar{L}^* > 50$	$\bar{L}^* \geq 75$
BFD-P (Luo and Rigg, 1986)	7.0	5.3	45.8	54.2	9.8
RIT-DuPont (Berns et al., 1991)	12.8	9.0	41.3	58.7	14.7
Leeds (Kim, 1997; Kim and Nobbs, 1997)	35.5	0.0	60.6	39.4	12.7
Witt (Witt, 1999)	6.7	0.0	40.7	59.3	20.3
RIT-DuPont-Ind. (Berns and Hou, 2010)	13.0	8.8	42.6	57.4	15.2
BIGC-T2-M (Huang, Wang, Liu, 2010)	8.0	0.0	35.1	64.9	22.8
BIGC-T2-SG (Huang, Wang, Liu, 2010)	12.6	0.0	35.2	64.8	24.7
BIGC-T2-G (Huang, Wang, Liu, 2010)	9.0	0.0	38.3	61.7	21.9
BIGC-T1-SG (Huang et al., 2012a)	10.0	0.0	37.0	63.0	12.3
Wang (Wang et al., 2012)	37.0	0.0	51.0	49.0	20.0
BIGC-S-SG (Huang et al., 2012b)	2.5	0.0	36.3	63.7	12.1
NCSU-Blue (Lee et al., 2011)	3.0	0.0	100.0	0.0	0.0
NCSU-Black (Shamey et al., 2014)	28.0	100.0	100.0	0.0	0.0

Table 1 - Percentages of color pairs with mainly lightness differences (column 2) and average CIELAB lightness $(\bar{L}^*)$ in four different intervals (columns 3-6), for each one of the 13 visual datasets considered in this paper.

Datasets (References)	ΔE_{00}	ΔE_{00}_M1	ΔE_{00}_M2	ΔE_{00}_M3	ΔE_{00}^0.70	F_C	$1/F_C$
BFD-P (Luo and Rigg, 1986)	29.6	*30.1*	**31.5**	*29.1*	**28.5**	0.928	1.077
RIT-DuPont (Berns et al., 1991)	19.5	*18.6*	*18.9*	*19.0*	**13.4**	0.800	1.249
Leeds (Kim, 1997; Kim and Nobbs, 1997)	19.2	**21.8**	**27.6**	*19.4*	*17.6*	0.799	1.252
Witt (Witt, 1999)	30.2	*28.2*	**27.4**	*30.6*	*28.6*	0.825	1.212
RIT-DuPont-Ind. (Berns and Hou, 2010)	23.1	*22.5*	*23.1*	*22.7*	**15.6**	0.872	1.146
BIGC-T2-M (Huang, Wang, Liu, 2010)	45.8	*45.8*	*46.0*	*46.1*	**38.2**	0.821	1.217
BIGC-T2-SG (Huang, Wang, Liu, 2010)	48.8	*47.9*	*46.8*	*48.6*	**39.6**	0.830	1.204
BIGC-T2-G (Huang, Wang, Liu, 2010)	50.3	*49.8*	*49.1*	*49.5*	**44.1**	0.817	1.224
BIGC-T1-SG (Huang et al., 2012a)	46.4	*46.1*	*45.9*	*46.0*	**36.1**	0.877	1.140
Wang (Wang et al., 2012)	18.3	**26.8**	**29.1**	*18.1*	**13.4**	0.673	1.486
BIGC-S-SG (Huang et al., 2012b)	29.4	*29.2*	*29.3*	*29.5*	**22.5**	0.830	1.204
NCSU-Blue (Lee et al., 2011)	21.2	*21.2*	*22.6*	*21.8*	*17.2*	0.612	1.633
NCSU-Black (Shamey et al., 2014)	25.9	*25.9*	*27.6*	*25.3*	*20.7*	0.567	1.762

Table 2 - *STRESS* values for 13 visual datasets (column 1) and five color-difference formulas (columns 2-6, see text). Cells filled with blue/yellow colors indicate better/worse performance than the one achieved by using the ΔE_{00} color-difference formula, while numbers in bold/italic font mean significant/non-significant differences with respect to ΔE_{00}, from specific F_C and $1/F_C$ critical values shown in last two columns assuming a significance level of 95%.

 DOI: 10.23738/ccsj.i102018.07

differences (Table 3).

From Table 3 it can be noted that none of the three CIEDE2000-modified color-difference formulas (ΔE_{00}_M1, ΔE_{00}_M2 and ΔE_{00}_M3) reported statistically significant better results than ΔE_{00} for any of the 13 datasets (i.e. there are no cells filled in blue color and numbers in bold font in columns 3-5 in Table 3).

Comparing Tables 2 and 3, it can be stated that the performance of all tested color-difference formulas (except ΔE_{00}_M3) with respect to the original color-difference formula ΔE_{00} is slightly worse for color pairs with almost only lightness differences (Table 3) than for complete datasets (Table 2). From Tables 2 and 3 it can be also added that color-difference formulas ΔE_{00}_M1 and ΔE_{00}_M3 (but not ΔE_{00}_M2) improved ΔE_{00} for most datasets, but such improvements never (except for 1 dataset) were statistically significant. Tables 2 and 3 show that for a majority of datasets the ΔE_{00}_M2 formula was significantly or non-significantly worse than the ΔE_{00} color-difference formula, which indicates that, beside the recommendation made by Berns (Berns, 2016), for current experimental datasets the replacement of the S_L function in ΔE_{00} by $S_L=1$ is not advisable.

2.3. COLOR PAIRS WITH AVERAGE L^* BELOW AND ABOVE 50

In this subsection our goal is to check the symmetry of the S_L function proposed by CIEDE2000 with respect to the assumed lightness of the background, $\bar{L}^*=50$ (see Eq. 2). Tables 4 and 5 show STRESS results found using color pairs in complete datasets with average L^* values ($\bar{L}^*$) below and above 50, respectively. The colors and fonts codes in the cells of Tables 4 and 5 are the same used in previous Tables 2 and 3.

It can be noted that Tables 4 and 5 discarded the use of the ΔE_{00}_M1 color-difference formula (and also the ΔE_{00} color-difference formula modified by the exponent 0.70), because this formula considers the lightness of the backgrounds in each dataset in place of ($\bar{L}^*$)=50 (Ho, 2006). Tables 4 and 5 also missed the NCSU-Blue (Lee et al., 2011) and NCSU-Black (Shamey et al., 2014) datasets because all color pairs in these datasets had ($\bar{L}^*$) values below 50 (see Table 1).

From Tables 4 and 5 we can see that both, ΔE_{00}_M2 and ΔE_{00}_M3 were never statistically significantly better than ΔE_{00}. It can be also noticed that the performance of these two CIEDE2000-modified color-difference formulas

Table 3 - *Idem to Table 2, but for color pairs with mainly lightness differences:* $|\Delta L^*/\Delta E^*_{ab}|>0.9$.

Datasets (References)	ΔE_{00}	ΔE_{00}_M1	ΔE_{00}_M2	ΔE_{00}_M3	ΔE_{00}^0.70	F_C	$1/F_C$
BFD-P (Luo and Rigg, 1986)	32.5	*31.4*	*31.1*	*30.3*	*32.2*	0.754	1.326
RIT-DuPont (Berns et al., 1991)	14.7	*13.8*	*17.6*	*13.4*	**10.2**	0.529	1.891
Leeds (Kim, 1997; Kim and Nobbs, 1997)	16.7	*19.5*	**25.4**	*16.9*	*14.9*	0.684	1.461
Witt (Witt, 1999)	29.7	*28.4*	*30.6*	*29.3*	*24.7*	0.463	2.161
RIT-DuPont-Ind. (Berns and Hou, 2010)	22.6	*22.2*	*25.3*	*21.9*	**14.2**	0.683	1.464
BIGC-T2-M (Huang, Wang, Liu, 2010)	55.0	*54.6*	*54.5*	*55.7*	*43.9*	0.488	2.049
BIGC-T2-SG (Huang, Wang, Liu, 2010)	38.6	*37.3*	*35.9*	*37.5*	*31.0*	0.586	1.706
BIGC-T2-G (Huang, Wang, Liu, 2010)	54.4	*53.7*	*52.7*	*51.4*	*45.6*	0.499	2.002
BIGC-T1-SG (Huang et al., 2012a)	45.1	*44.2*	*43.2*	*44.5*	**34.2**	0.657	1.523
Wang (Wang et al., 2012)	14.1	**27.7**	**25.1**	*13.3*	*10.5*	0.515	1.942
BIGC-S-SG (Huang et al., 2012b)	28.8	*29.7*	*32.8*	*28.0*	*21.7*	0.269	3.718
NCSU-Blue (Lee et al., 2011)	21.8	*21.8*	*22.9*	*22.5*	*14.8*	0.002	647.8
NCSU-Black (Shamey et al., 2014)	22.6	*22.9*	*23.7*	*20.3*	*22.8*	0.321	3.115

Table 4 - *STRESS values for color pairs with average* L^* *values above 50, considering 3 color-difference formulas and 11 visual datasets. The colors of the cells and the fonts for numbers in current Table 4 follow the same codes used in previous Tables 2 and 3 (see text).*

Datasets (References)	ΔE_{00}	ΔE_{00}_M2	ΔE_{00}_M3	F_C	$1/F_C$
BFD-P (Luo and Rigg, 1986)	26.9	**31.4**	*26.5*	0.904	1.106
RIT-DuPont (Berns et al., 1991)	18.7	*19.0*	*19.2*	0.747	1.338
Leeds (Kim, 1997; Kim and Nobbs, 1997)	18.9	**29.7**	*18.9*	0.698	1.433
Witt (Witt, 1999)	30.0	*27.0*	*30.2*	0.779	1.284
RIT-DuPont-Ind. (Berns and Hou, 2010)	23.1	*23.8*	*23.5*	0.835	1.198
BIGC-T2-M (Huang, Wang, Liu, 2010)	42.8	*43.3*	*42.9*	0.783	1.277
BIGC-T2-SG (Huang, Wang, Liu, 2010)	39.0	*38.4*	*38.6*	0.793	1.260
BIGC-T2-G (Huang, Wang, Liu, 2010)	43.5	*43.3*	*43.4*	0.773	1.294
BIGC-T1-SG (Huang et al., 2012a)	44.3	*44.1*	*44.3*	0.847	1.180
Wang (Wang et al., 2012)	12.8	**27.0**	*12.7*	0.564	1.773
BIGC-S-SG (Huang et al., 2012b)	29.7	*29.5*	*29.8*	0.792	1.263

Table 5 - *Idem to Table 4, but for color pairs with average* L^* *values below 50.*

Datasets (References)	ΔE_{00}	ΔE_{00}_M2	ΔE_{00}_M3	F_C	$1/F_C$
BFD-P (Luo and Rigg, 1986)	31.6	*31.5*	*31.2*	0.896	1.116
RIT-DuPont (Berns et al., 1991)	19.4	*17.7*	*18.1*	0.706	1.416
Leeds (Kim, 1997; Kim and Nobbs, 1997)	17.9	**21.1**	*19.2*	0.749	1.335
Witt (Witt, 1999)	17.1	*15.7*	*16.3*	0.739	1.353
RIT-DuPont-Ind. (Berns and Hou, 2010)	21.7	*20.6*	*20.8*	0.811	1.233
BIGC-T2-M (Huang, Wang, Liu, 2010)	48.7	*48.7*	*48.6*	0.716	1.396
BIGC-T2-SG (Huang, Wang, Liu, 2010)	56.1	*54.9*	*55.5*	0.730	1.370
BIGC-T2-G (Huang, Wang, Liu, 2010)	54.8	*52.9*	*53.6*	0.720	1.388
BIGC-T1-SG (Huang et al., 2012a)	49.6	*48.7*	*48.4*	0.805	1.242
Wang (Wang et al., 2012)	19.0	*22.9*	*20.7*	0.571	1.752
BIGC-S-SG (Huang et al., 2012b)	28.9	*28.9*	*28.9*	0.733	1.363

with respect to ΔE_{00} is better for color pairs with $\overline{L}^*$ values below 50 (Table 5) than for color pairs with $\overline{L}^*$ values above 50 (Table 4), but this result must be interpreted with prudence, because the number of available color pairs in the range $\overline{L}^*\leq 25$ is very small (see Table 1), and this difference is not statistically significant (see subsection 2.5).

2.4. NORMALIZED SCATTER PLOTS FOR COLOR PAIRS WITH MAINLY LIGHTNESS DIFFERENCES

Figures 1 and 2 show the normalized ratios $\Delta E^*_{ab}/\Delta V$ for color pairs with $|\Delta L^*/\Delta E^*_{ab}|>0.9$ (i.e. color pairs with mainly lightness differences) against average L^* values below and above 50, respectively, for different visual datasets. Specifically, we considered 11 visual datasets, discarding from the 13 initial ones (Table 1) the two datasets with color pairs in only one region of the color space (Lee et al., 2001; Shamey et al., 2014). The normalization used in Figures 1 and 2 was to divide the mentioned ratios by its average in each individual dataset, as made by Berns (Berns, 2016). Figures 1 and 2 also show the two branches of the S_L (V-shaped) function, corresponding to the predictions made by the original ΔE_{00} color-difference formula (ISO/CIE, 2014; CIE, 2001; Luo, Cui and Rigg, 2001). As in plots reported by Berns (Berns, 2016), Figures 1 and 2 show a considerable scatter. Therefore, in general, it cannot be stated that the S_L function (Eq. 2) proposed by ΔE_{00} (Eq. 1) is a good predictor of experimental results for color pairs with almost only lightness differences in most currently available visual datasets. Figure 1 also shows that there are few pairs in the range $L^*\leq 25$ (see Table 1), which claims for future studies using a higher number of dark color pairs.

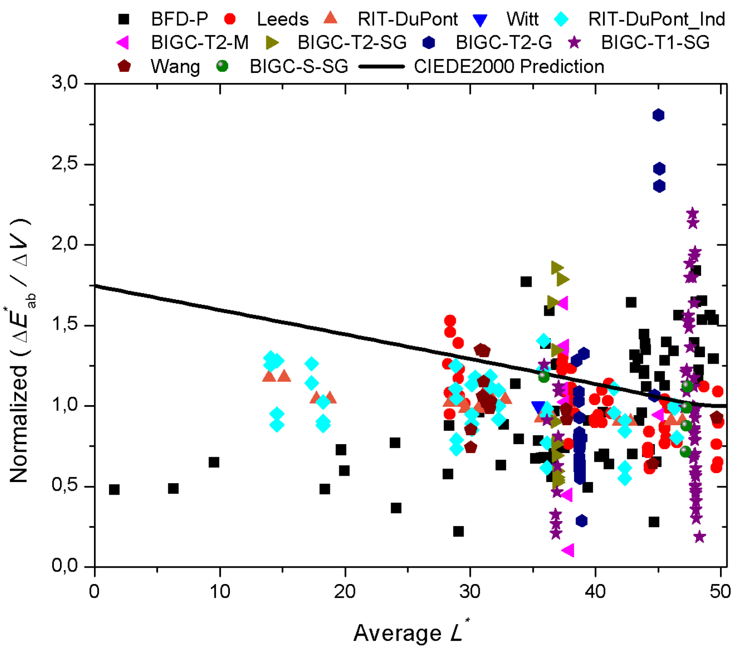

Figure 1 - *Normalized ratios* $\Delta E^*_{ab}/\Delta V$ *for experimental color pairs with* $|\Delta L^*/\Delta E^*_{ab}|>0.9$ *(i.e. color pairs with mainly lightness differences) in 11 visual datasets, against average* L^* *values of color pairs in the range 0-50. The predictions made by the* ΔE_{00} *color-difference formula are indicated by the black line.*

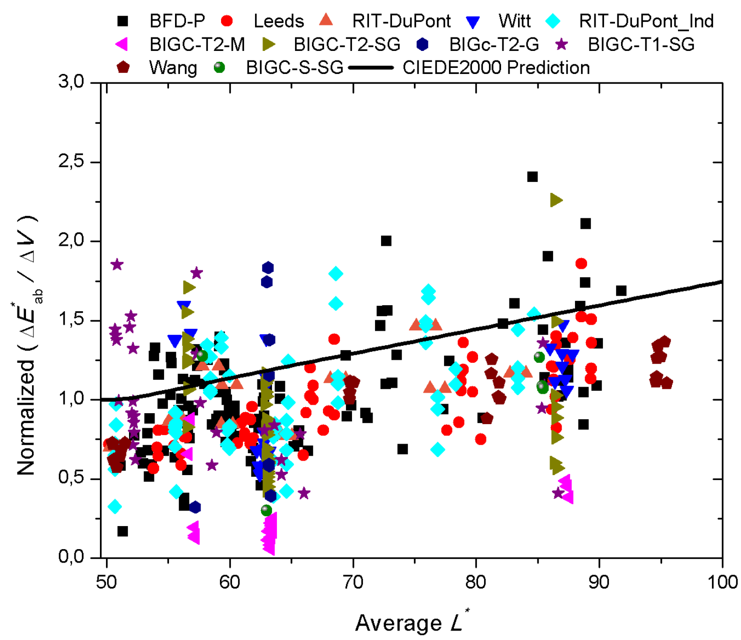

Figure 2 - *Idem to Figure 1, but for color pairs with average* L^* *values in the range 50-100.*

2.5. SUMMARY OF RESULTS FOR COMPLETE DATASETS AND SUBSETS

Table 6 summarizes the results shown in previous Tables 2-5, indicating the number of datasets (from a total of 13 or 11, depending on the row considered in such Table) with better or worse results than those achieved by the original ΔE_{00} color-difference formula, considering also statistically and not statistically significant differences at 95% confidence level. Note that, for the three CIEDE2000-modified formulas (ΔE_{00}_M1, ΔE_{00}_M2 and ΔE_{00}_M3) statistically significant better results than those found by using ΔE_{00} were found only for one dataset (Witt, 1999). However, the ΔE_{00} color-difference formula modified by an exponent 0.7 (Huang et al., 2015) produced very good results for all complete datasets (i.e. statistically significant or non-significant improvements for all datasets), and also good results for the subsets of color pairs with mainly lightness differences. In general, we can note that highest values in Table 6 are located in column 4, which means that for most datasets the CIEDE2000-modified formulas tested in the current paper were better, but, unfortunately, not statistically significantly better, than the original ΔE_{00} color-difference formula.

With respect to the symmetry around $\bar{L}^*=50$, it can be noted in Table 6 that the CIEDE2000-modified formulas ΔE_{00}_M2 and ΔE_{00}_M3 performed better than the original ΔE_{00} formula in a majority of datasets for color pairs with average L^* below 50, but not for color pairs with average L^* above 50. However, from Wilcoxon sign rank test, the medians of STRESS values for color pairs with average L^* values above and below 50 (Tables 4 and 5) were not statistically significantly different for ΔE_{00} (p=0.240), ΔE_{00}_M2 (p=0.966), and ΔE_{00}_M3 (p=0.240).

3. CONCLUSIONS

The analyses presented in the current paper are complementary to those reported in Melgosa et al., 2017. From most currently available experimental datasets, our analyses do not allow a successful alternative proposal to the S_L function proposed by ΔE_{00}, because in most cases the improvements achieved by the candidate formulas are not statistically significant. In particular, our results show that replacing the S_L function proposed in ΔE_{00} by $S_L=1$, as done by CIELAB and CIE94 (CIE, 2004), and also recently suggested by Berns (Berns, 2016), is not a good choice. However, the replacement of CIELAB lightness L^* by another lightness function, shown by the Eqs. 4 and 5, based on Whittle formula (Whittle, 1992), leads to promising results. Figures 1 and 2 also indicate that the S_L function proposed by ΔE_{00} is not a satisfactory definitive result. However, it must be added that ΔE_{00} was recommended for a specific set of viewing conditions, the so-called 'reference conditions' (ISO/CIE, 2014; CIE, 2001; Luo, Cui and Rigg, 2001), which are not identical to those employed in all the visual datasets considered in the current paper (Table 1). We hope that advances on new color spaces, in particular those with physiological basis, as well as a sounder knowledge of the influence of specific viewing conditions (parametric factors) on perceived color differences will lead to future color-difference formulas with improved performance.

Table 6 - *Number of datasets or subsets with different kind of differences (columns 3-6) between several CIEDE2000-modified color-difference formulas (see text) and the original ΔE_{00} color-difference formula.*

Datasets	Two color-difference formulas under comparison	Significantly Better	Non-Significantly Better	Significantly Worse	Non-Significantly Worse
Complete Datasets	$\Delta E_{00}\_M1\ vs.\ \Delta E_{00}$	0	10	2	1
	$\Delta E_{00}\_M2\ vs.\ \Delta E_{00}$	1	5	3	4
	$\Delta E_{00}\_M3\ vs.\ \Delta E_{00}$	0	8	0	5
	$\Delta E_{00}\hat{}0.7\ vs.\ \Delta E_{00}$	9	4	0	0
Only pairs with $\lvert \Delta L^*/\Delta E^*_{ab} \rvert > 0.9$	$\Delta E_{00}\_M1\ vs.\ \Delta E_{00}$	0	8	1	4
	$\Delta E_{00}\_M2\ vs.\ \Delta E_{00}$	0	5	2	6
	$\Delta E_{00}\_M3\ vs.\ \Delta E_{00}$	0	10	0	3
	$\Delta E_{00}\hat{}0.7\ vs.\ \Delta E_{00}$	3	9	0	1
Only pairs with average L^* > 50	$\Delta E_{00}\_M2\ vs.\ \Delta E_{00}$	0	5	3	3
	$\Delta E_{00}\_M3\ vs.\ \Delta E_{00}$	0	5	0	6
Only pairs with average L^* < 50	$\Delta E_{00}\_M2\ vs.\ \Delta E_{00}$	0	8	1	2
	$\Delta E_{00}\_M3\ vs.\ \Delta E_{00}$	0	8	0	3

CONFLICTS OF INTEREST

The authors declare no conflict of interest affecting the results reported in this paper.

FUNDING

This work was supported by Ministry of Economy and Competitiveness of the Government of Spain, research projects FIS2013-40661-P and FIS2016-80983-P, with support from European Regional Development Fund (ERDF).

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